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A novel stochastic ten non-polynomial cubic splines method for heat equations with noise term

Aisha F. Fareed [a,b], Ahmed G. Khattab [b], Mourad S. Semary [b]

[a] Department of Electrical Engineering, College of Engineering, Prince Sattam bin Abdulaziz University, Al Kharj 16278, Saudi Arabia
[b] Department of Basic Engineering Sciences, Benha Faculty of Engineering, Benha University, Benha, Egypt

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ABSTRACT

In this paper, a new numerical method for solving a class of stochastic partial differential equations is presented. The proposed method is based on a non-polynomial cubic spline algorithm with an $O(h^4)$ local truncation error. The new approach has the advantage of simultaneously including ten non-polynomial cubic spline schemes. The proposed approach is also tested on three stochastic heat equations. The current results are compared to solutions obtained using the B-spline wavelet approximation method. The provided solutions demonstrate the proposed method's accuracy and applicability.

1. Introduction

Stochastic partial differential equations (SPDEs) are frequently utilized to describe highly complex phenomena that exhibit both deterministic and stochastic behavior in physics (diffusion theory, growth of crystal surfaces),[1] engineering (thermal noise, filtering and image analysis),[2,3] finance (option pricing),[4,5] and biology (population dynamics).[6] Due to the mix of temporal and spatial dynamics, as well as the existence of stochastic noise, solving SPDEs is a difficult mission.

Various analytical and numerical algorithms are developed to solve SPDEs such as variation of parameter method,[2] homotopy analysis method,[7] homotopy perturbation method,[8,9] Wiener-Chaos expansion,[9,10] spectral methods,[11,12] Monte-Carlo Method,[13] integro-quadratic spline approach,[14] discrete Temimi-Ansari method,[15,16] finite difference method,[17–19] finite element method,[19,20] collocation method,[21,22] theta method,[23] Tau-finite difference method,[24] Galerkin method,[25] spectral Galerkin method with backward Euler method,[26] collocation method with operational matrix method[27]… etc. Also, there are some references for the numerical solutions of SPDEs,[28–42] the researcher should read them.

Non-polynomial splines are used to solve a wide range of partial differential equations, including parabolic,[43] hyperbolic,[44] Burgers' Fisher,[45] fractional Burger and coupled Burgers' equations.[46] The ten non-polynomial cubic spline approach,[47] which we previously utilized to solve various classes of Fredholm Integral equations and achieved excellent results, has been developed in the current research to investigate the numerical solutions of the following SPDE,[48]

$$\begin{aligned} &u_t(x,\ t) = f_2 u_{xx}(x,\ t) + f_1 u_x(x,\ t) + f_0 u(x,\ t) + \rho\ u(x,\ t)\ n(t),\\ &u(x,\ 0) = g(x),\ x \in [a,\ b],\\ &u(a,\ t) = c_1,\ u(b,\ t) = c_2,\ t \geq 0, \end{aligned} \tag{1.1}$$

where $u(x, t)$ is the unknown function, f_0, f_1, f_2 and ρ are constants and $n(t)$ is the Gaussian white noise which relation with the Wiener process $\mathbb{W}(t)$ is:

$$n(t) = \xi \frac{d\ \mathbb{W}(t)}{dt}, \tag{1.2}$$

with zero expectation and a variance equal ξ^2.

This work aims to improve the ten non-polynomial cubic spline to solve one-dimensional stochastic partial differential equations (SPDEs) and obtain outstanding performance[36] with a local truncation error of $O(h^4)$. The suggested approach has the following advantages: it includes ten non-polynomial cubic spline schemes at the same time, and it investigates an approximation for the function $u(x, t)$ as well as its derivatives. To put the proposed method to the test, we examine its stability and solve three different stochastic heat equation models. These examples demonstrate the suggested method's accuracy when compared to a numerical approach based on B-spline wavelet approximations.[48]

This paper is structured as follows: The stochastic ten non-polynomial cubic spline approach is introduced in Section 2. The errors estimation and stability analysis are considered in Section 3. Sections 4 and 5 provide the numerical results of the three cases presented,

E-mail address: a.fareed@psau.edu.sa (A.F. Fareed).

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